Uncertainty Quantification Problems in Tsunami Modeling Using Reduced Order Models

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Two UQ Problems

(P1) **Tsunami source inversion problem**
estimate the posterior probability of the earthquake slip distribution, given measurements of the tsunami at a few scattered points.

(P2) **Probabilistic Tsunami Hazard Assessment (PTHA)**
compute the probability of inundation at given spatial points of interest, usually geographical locations of coastal communities.

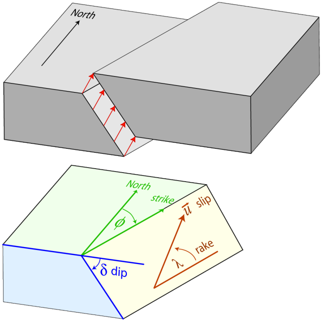
Tsunami Propagation

- Shallow water equation with source terms.

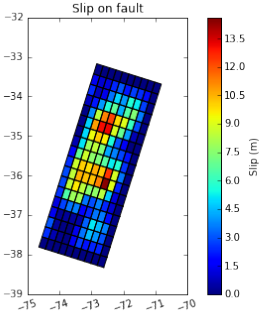
$$\begin{bmatrix} h \\ hu \end{bmatrix}_t + \begin{bmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \end{bmatrix}_x = \begin{bmatrix} 0 \\ -ghB_x \end{bmatrix}$$

  h: surface height, u: velocity, B: bathymetry, g: gravity constant
- **Initial condition**: given by the slip on the fault (subdivided into rectangles).

Rectangular fault

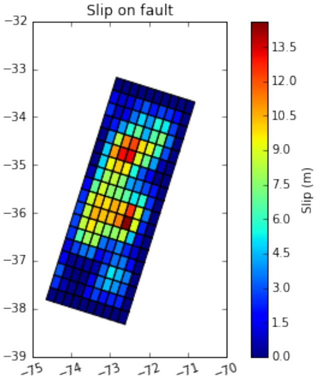


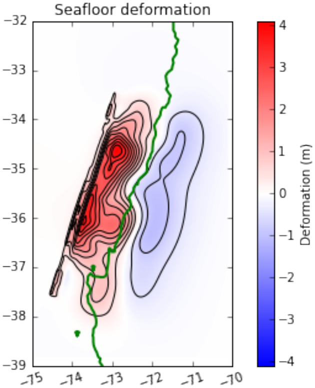
Input parameters: slip on subfaults



Tsunami Propagation (cont'd)

- The slip distribution on the fault then deforms the seafloor.
- **Okada Model** is used to compute the seafloor deformation.

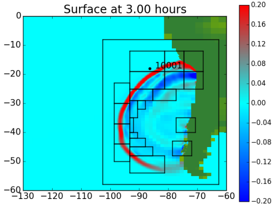


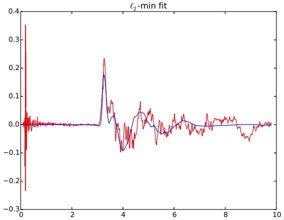


- The seafloor deformation is then used as the vertical displacement of the ocean surface, yielding the initial condition that causes gravity waves.

Tsunami Source Inversion Problem

- Source inversion using **DART Buoy data** (organized in $\mathbf{d}$)





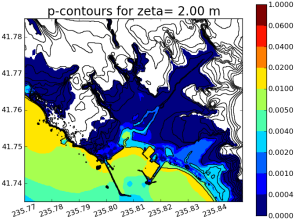
- **Bayesian Inverse Problem**: Let f denote the numerical solution operator (GeoClaw). We want to sample the posterior:

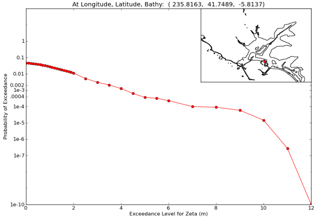
$$p_{\text{posterior}}(\mathbf{s}) \sim p_{\text{prior}}(\mathbf{s}) \exp\left(-\frac{1}{2}\left\|f(\mathbf{s}) - \mathbf{d}\right\|_2^2\right)$$

- MCMC methods: e.g., affine-invariant ensemble sampler

Probabilistic Tsunami Hazard Assessment (PTHA)

- Given a probability distribution over the slips $\mathbf{s}$, estimate the inundation probability at points of interest.
- Create a probabilistic map of events for coastal communities





- Given a point $\mathbf{x}$, and given inundation level ζ, compute the probability **maximum surface displacement** η_{max} exceeds ζ:

$$P[\eta_{\text{max}} > \zeta; \mathbf{x}]$$

(e.g. using importance sampling)

Inverse Problem and the PTHA

- Source inversion yields a posterior distribution on the slips $\mathbf{s}$
- $p_{\text{posterior}}(\mathbf{s})$ can be used to update the exceedance probability for PTHA